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STATISTICAL TRANSFER COEFFICIENTS OF HOMOGENEOUS VELOCITY AND TEMPERATURE FIELDS

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Abstract—Based both on the equations for mean squares of velocity and temperature fluctuations and on the model ones for appropriate field vorticity, consideration is made of degeneration of homogeneous and nonisotropic velocity and temperature fields. Asymptotic relations have been checked experimentally.

NOMENCLATURE

τ,	time;
q^2,	$= \bar{u}_i^2$, turbulence intensity;
Δ_ξ,	$= \dfrac{\partial^2}{\partial \xi^2}$, Laplace operator in ξ-space;
ξ_i,	vector component for distance between two points;
$\overline{u_i u_i'}$,	two-point velocity correlation;
ν,	coefficient of kinematic viscosity;
κ,	thermal diffusivity;
x_i,	Cartesian system of coordinates;
$\overline{tt'}$,	two-point temperature correlation;
Pr,	$= \dfrac{\nu}{\kappa}$, Prandtl number.

1. INTRODUCTION

As is known, the model of homogeneous and nonisotropic turbulence gives a more strict approach to real turbulence, as compared to the simplest and most thoroughly studied one for homogeneous and isotropic turbulence. The mathematical techniques of correlation tensors of axisymmetric turbulence, being the simplest form of homogeneous nonisotropic turbulence, were developed by Batchelor [1] and Chandrasekhar [2] on the same level as that of isotropic turbulence. However, the use of these techniques to approximately describe real turbulent flows involves difficulties because the problems on homogeneous and nonisotropic turbulence transfer are not yet worked out.

In the present work an attempt is made to employ the model equations of velocity and temperature field vorticity as well as those of fluctuation "intensity" for approximate description of velocity and temperature fields at homogeneous nonisotropic turbulence. In this case the authors have tried to answer the following two questions: (1) May the results of the known exact relations for decay of vector and scalar isotropic fields be used to describe homogeneous and nonisotropic fields? (2) Are the transfer coefficients of the model equations for vorticity constants and if not, then which combinations of the above coefficients and the turbulent and Peclet numbers are constants?

2. ANALYTICAL CONSIDERATION OF STATISTICAL TRANSFER COEFFICIENTS AT UNIFORM TURBULENCE

The equations for the mean square intensity of fluctuation velocity and vorticity at homogeneous turbulence may be written as:

$$\frac{dq^2}{d\tau} + 10\nu D_u = 0 \tag{2.1}$$

$$\frac{d}{d\tau} D_u + \frac{7}{3\sqrt{3}} (S_\nu + S_u) \cdot D_u^{3/2} = 0 \tag{2.2}$$

where $D_u = \frac{1}{3}(-\Delta_\xi \overline{u_i u_i'})_{\xi=0}$ is the vorticity of a velocity field; the coefficients S_ν and S_u are the dimensionless quantities composed of differential operators with respect to a variable ξ of two-point velocity correlations:

$$S_\nu = \frac{6\sqrt{15}}{7} \nu \frac{(-\Delta_\xi \Delta_\xi \overline{u_i u_i'})_{\xi=0}}{(-\Delta_\xi \overline{u_i u_i'})_{\xi=0}^{3/2}} \tag{2.3}$$

$$S_u = -\frac{6\sqrt{15}}{7} \frac{\left(-\dfrac{\partial}{\partial \xi_j} \Delta_\xi \overline{u_j u_i u_i'}\right)_{\xi=0}}{(-\Delta_\xi \overline{u_i u_i'})_{\xi=0}^{3/2}}. \tag{2.4}$$

For homogeneous and isotropic turbulence, equations (2.1) and (2.2) assume the well-known form [3] where the vorticity D_u is expressed in terms of Taylor's dissipation scale according to the obvious identity

$$D_u^* = \frac{3\bar{u}^2}{\lambda_u^2}$$

where the asterisk denotes that the function is considered at isotropic turbulence, and the coefficients S_ν and S_u admit the well-known form:

$$S_\nu^* = 2\nu \frac{\overline{\left(\dfrac{\partial^2 u_1}{\partial x_1^2}\right)^2}}{\overline{\left(\dfrac{\partial u_1}{\partial x_1}\right)^2}^{3/2}} \tag{2.3*}$$

$$S_u^* = \frac{\overline{\left(\dfrac{\partial u_1}{\partial x_1}\right)^3}}{\overline{\left(\dfrac{\partial u_1}{\partial x_1}\right)^2}^{3/2}} \tag{2.4*}$$

where u_1 is the velocity fluctuation in the x_1-direction.

Similar to equations (2.1) and (2.2), the equations may be written for the mean square of fluctuations and "dissipation" (smearing) function of a uniform temperature field

$$\frac{d}{d\tau}\bar{t}^2+12\kappa D_t=0 \tag{2.5}$$

$$\frac{d}{d\tau}D_t+\frac{5}{\sqrt{3}}(\tfrac{2}{3}S_\kappa+S_t)D_u^{1/2}D_t=0 \tag{2.6}$$

where $D_t=\frac{1}{6}(-\Delta_\xi\overline{tt'})_{\xi=0}$ is the smearing function of a temperature field, the coefficients S_κ and S_t mean the dimensionless functions of differential operators with respect to ξ of two-point velocity and temperature correlations:

$$S_\kappa=\frac{3\sqrt{15}}{5}\kappa\frac{(-\Delta_\xi\Delta_\xi\overline{tt'})_{\xi=0}}{(-\Delta_\xi\overline{tt'})_{\xi=0}(-\Delta_\xi\overline{u_iu_i'})^{1/2}_{\xi=0}} \tag{2.7}$$

$$S_t=-\frac{2\sqrt{15}}{5}\frac{\left(-\frac{\partial}{\partial\xi_j}\overline{u_itt'}\right)_{\xi=0}}{(-\Delta_\xi\overline{tt'})_{\xi=0}(-\Delta_\xi\overline{u_iu_i'})^{1/2}_{\xi=0}}.$$

At homogeneous and isotropic turbulence, the function D_t is expressed in terms of a dissipation scale of a temperature field (thermal microscale) in the form:

$$D_t^*=\frac{\bar{t}^2}{\lambda_t^2}$$

and the coefficients $S_æ$ and S_t take the form:

$$S_\kappa^*=\kappa\frac{\overline{\left(\frac{\partial^2t}{\partial x_1^2}\right)^2}}{\overline{\left(\frac{\partial t}{\partial x_1}\right)^2}\cdot\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2}^{1/2}} \tag{2.7*}$$

$$S_t^*=\frac{\overline{\left(\frac{\partial t}{\partial x_1}\right)^2\cdot\left(\frac{\partial u_1}{\partial x_1}\right)}}{\overline{\left(\frac{\partial t}{\partial x_1}\right)^2}\,\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2}^{1/2}}. \tag{2.8*}$$

Applicability of equations (2.1)–(2.2) and (2.5)–(2.6) to describe homogeneous velocity and temperature fields is established by a knowledge of the coefficients S_v, S_u and $S_æ$, S_t as the functions of the Reynolds and Peclet numbers

$$R_l=\frac{ql_u}{\nu},\quad P_l=\frac{ql_t}{\kappa}$$

where l_u and l_t are some length scales of velocity and temperature fields, respectively. As far as the authors know, the functional relations of the above coefficients for homogeneous and nonisotropic fields in the forms (2.3)–(2.4) and (2.7)–(2.8) were not earlier determined. There are nevertheless a great number of works [4–6] covering the information on the coefficient S_u^* for an isotropic field. The data on the coefficient S_v^* are given in [3]. In the equation for D_t the appropriate coefficients for an isotropic temperature field are less studied. The authors know only the works by Yaglom [7] and Wyngaard [8], in which consideration is made of the coefficient S_t^*; the authors of the present work are not familiar with works devoted to determination of $S_æ^*$.

The functional dependences of S_v^*, S_u^* and $S_æ^*$, S_t^* upon the Reynolds and Peclet numbers may be obtained only at limiting values of these determining parameters. Indeed, at $R_l \ll 1$ and $P_l \ll 1$ diffusional terms may be neglected in equations (2.4*) and (2.8*). Then, we shall have the following two pairs of the equations for velocity and temperature fields, respectively:

$$\frac{d}{d\tau}q^2+10\nu\cdot\frac{q^2}{\lambda_u^2}=0$$

$$\frac{d}{d\tau}\lambda_u^2+\nu\left(10-\frac{7}{3\sqrt{3}}S_v^*\cdot R_\lambda\right)=0$$

$$\frac{d\bar{t}^2}{d\tau}+12\kappa\cdot\frac{\bar{t}^2}{\lambda_t^2}=0$$

$$\frac{d}{d\tau}\lambda_t^2+\kappa\left(12-\frac{10}{3\sqrt{3}}S_æ\frac{P_\lambda^2}{R_\lambda}\cdot\frac{1}{Pr}\right)=0$$

$R_\lambda=q\lambda_u/\nu$ and $P_\lambda=q\lambda_t/\kappa$ are the "microscale" Reynolds and Peclet numbers, respectively. The solutions of these pairs of the equations at

$$S_v^*\cdot R_\lambda=S_{u\,\mathrm{dis}}=\text{const and } S_\kappa\cdot\frac{P_\lambda^2}{R_\lambda}\cdot\frac{1}{Pr}=S_{t\,\mathrm{dis}}=\text{const}$$

admit the form:

$$q^2(\lambda_u^2)^{1/a}=\text{const} \tag{2.9}$$

$$t^2\cdot(\lambda_t^2)^{1/b}=\text{const} \tag{2.10}$$

where

$$a=\frac{7}{30\sqrt{3}}S_{u\,\mathrm{dis}}-1;\quad b=\frac{5}{18\sqrt{3}}\cdot S_{t\,\mathrm{dis}}-1.$$

Proceeding from Loitsyansky [9] and Corrsin [10] invariants, in equations (2.9) and (2.10) $a = 2/5$ and $b = 2/3$ should be assumed. In this case we shall have

$$S_{u\,\mathrm{dis}}^*=S_{t\,\mathrm{dis}}^*=6\sqrt{3}. \tag{2.11}$$

Thus, at very low values of the Reynolds and Peclet numbers the coefficients $S_{u\,\mathrm{dis}}^*$ and $S_{t\,\mathrm{dis}}^*$ for isotropic velocity and temperature fields are constants, i.e. the coefficients S_v^* and S_κ^* are the functions of the Reynolds and Peclet numbers

$$S_v^*=\frac{6\sqrt{3}}{R_\lambda},\quad S_\kappa=6\sqrt{3}\frac{R_\lambda}{P_\lambda^2}\cdot Pr.$$

At very high values of the Reynolds and Peclet numbers, equations (2.1)–(2.2) and (2.5)–(2.6) in terms of large scale turbulence may be written as:

$$\frac{dq}{d\tau}+5\frac{q^3}{L_u}=0$$

$$\frac{dL_u}{d\tau}+5\left[3-\frac{7}{15\sqrt{3}}(S_v^*+S_u^*)\cdot R_L^{1/2}\right]q=0$$

$$\frac{d\bar{t}^2}{d\tau}+12q\frac{\bar{t}^2}{L_t}=0$$

$$\frac{dL_t}{d\tau}+12q\left\{1-\frac{5}{12}\left[\frac{1}{\sqrt{3}}(\tfrac{2}{3}S_\kappa^*+S_t^*)-\frac{1}{R_L^{1/2}}\right]\right\}=0$$

where

$$L_u = \frac{q^3}{\nu D_u^*} \quad \text{and} \quad L_t = \frac{q\bar{t}^2}{\kappa D_t^*}$$

are the scales of decay of velocity and temperature fields; $R_L = qL_u/\nu$ and $P_L = qL_t/\kappa$ are the "macroscale" Reynolds and Peclet numbers, respectively. If the following relations

$$(S_v^* + S_u^*)R_L^{1/2} = S_{u\,\mathrm{dis}}^* - S_{u\,\mathrm{dif}}^* = \mathrm{const}$$

$$\frac{1}{Pr}(\tfrac{2}{3}S_\kappa^* + S_t^*)\frac{P_L}{R_L^{1/2}} - \frac{\sqrt{3}}{Pr}\frac{P_L}{R_L} = \tfrac{2}{3}S_{t\,\mathrm{dis}} - S_{t\,\mathrm{dif}} = \mathrm{const}$$

are assumed to be valid, then the solutions of the above pairs of the equations are of the form:

$$q^2 L_u^{1/a} = \mathrm{const}; \qquad \bar{t}^2 L_t^{1/b} = \mathrm{const}$$

where

$$a = \frac{7}{30\sqrt{3}}(S_{u\,\mathrm{dis}}^* - S_{u\,\mathrm{dif}}^*) - \tfrac{3}{2}$$

$$b = \frac{5}{12\sqrt{3}}\left(\tfrac{2}{3}S_{t\,\mathrm{dis}}^* - S_{t\,\mathrm{dif}}^* - \frac{\sqrt{3}}{Pr}\cdot\frac{P_L}{R_L}\right) - 1.$$

That the above equalities should satisfy the Loitsyansky and Corrsin invariants, $a = 1/5$, $b = 1/3$ should be assumed. As a result the relations for dissipative and diffusional coefficients:

$$S_{u\,\mathrm{dis}}^* - S_{u\,\mathrm{dif}} = \frac{51\sqrt{3}}{7} \tag{2.12}$$

$$\tfrac{2}{3}S_{t\,\mathrm{dis}}^* - S_{t\,\mathrm{dif}}^* - \frac{\sqrt{3}}{\sqrt{Pr}}\frac{P_L}{R_L} = \frac{16\sqrt{3}}{5} \tag{2.13}$$

may be obtained. These relations show that at large values of the Reynolds and Peclet numbers, not the coefficients S_u^*, S_v^* and S_t^*, S_κ^* which enter equations (2.2) and (2.6) and are often assumed to be constants [3], are constants but their combinations $S_{u\,\mathrm{dis}}^*$, $S_{u\,\mathrm{dif}}^*$ and $S_{t\,\mathrm{dis}}^*$, $S_{t\,\mathrm{dif}}^*$ in the L.H.S. of equations (2.12) and (2.13).

Asymptotic relations (2.11), (2.12) and (2.13) are valid for homogeneous and isotropic velocity and temperature fields. The analysis, similar to the above one, may not be made of homogeneous but nonisotropic fields, as the laws of degeneration of such fields are insufficiently studied. Direct experimental check of the asymptotic relations obtained is therefore advisable.

In the experiment under consideration statistical transfer coefficients $S_{u\,\mathrm{dis}}$, $S_{u\,\mathrm{dif}}$, $S_{t\,\mathrm{dis}}$ and $S_{t\,\mathrm{dif}}$ are measured over a wide range of the Reynolds and Peclet numbers to check, whether there exists an agreement between the values of the coefficients from asymptotic relations (2.12) and (2.13) and those found from direct experiments on homogeneous and nonisotropic turbulence.

The coefficients $S_{u\,\mathrm{dis}}$, $S_{u\,\mathrm{dif}}$, $S_{t\,\mathrm{dis}}$, $S_{t\,\mathrm{dif}}$, written in terms of the correlation functions, are of the form:

$$S_{u\,\mathrm{dis}} = \frac{30\sqrt{3}}{7}q^2\frac{(-\Delta_\xi\Delta_\xi\overline{u_i u_i'})_{\xi=0}}{(-\Delta_\xi\overline{u_i u_i'})_{\xi=0}^2} \tag{2.14}$$

$$S_{u\,\mathrm{dif}} = \frac{-30\sqrt{3}}{7}\cdot\frac{q^2}{\nu}\cdot\frac{\left(-\dfrac{\partial}{\partial\xi_i}\Delta\xi\overline{u_j u_i u_i'}\right)_{\xi=0}}{(-\Delta_\xi\overline{u_i u_i'})_{\xi=0}^2} \tag{2.15}$$

$$S_{t\,\mathrm{dis}} = \frac{18\sqrt{3}}{5}\bar{t}^2\frac{(-\Delta_\xi\Delta_\xi\overline{tt'})_{\xi=0}}{(-\Delta_\xi\overline{tt'})_{\xi=0}^2} \tag{2.16}$$

$$S_{t\,\mathrm{dif}} = \frac{12\sqrt{3}}{5}\frac{\bar{t}^2}{\kappa}\frac{\left(-\dfrac{\partial}{\partial\xi_i}\Delta\xi\overline{u_i tt'}\right)_{\xi=0}}{(-\Delta_\xi\overline{tt'})_{\xi=0}^2}. \tag{2.17}$$

3. EXPERIMENTAL ARRANGEMENT AND TECHNIQUES

From relations (2.14)–(2.17) it follows that for experimental check of statistical transfer coefficients measurement should be performed of two-point correlations functions of temperature and velocity in three directions

$$\overline{u_i u_i'}(\xi),\quad \overline{u_j u_i u_i'}(\xi),\quad \overline{tt'}(\xi),\quad \overline{u_i tt'}(\xi)$$

and then calculation should be made of the derivatives of appropriate orders (up to the fourth order inclusive) of the correlation functions. The simplest form of uniform nonisotropic turbulence, i.e. axisymmetric turbulence, was employed in such an experiment.

Experiments were carried out in the closed-type wind tunnel with a round cross-section, 1.25 m in dia. The homogeneity of the simplest statistical characteristics of a velocity field U_1, $\overline{u_1 u_2}$, $\bar{u}_1^2$, $\bar{u}_2^2$, $\bar{u}_3^2$, $\bar{u}_1^3$ was achieved by placing a coarse-cell grid (turbulence generator) before the confuser and some fine-cell grids, in different cross-sections of the "return" duct. The turbulence generator is made of duralium 0.021 m dia tubes. Here, in the test section of the wind tunnel the flow is developed with the following characteristics:

$$\bar{U}_1 = 4\text{–}24\,\mathrm{m/s};\quad \frac{(\bar{u}_1^2)^{1/2}}{U_1} = 0.007;\quad \frac{\bar{u}_2^2}{\bar{u}_1^2} = \frac{\bar{u}_3^2}{\bar{u}_1^2} = 1.7;$$

$$\frac{\bar{u}_1^3}{(\bar{u}_1^2)^{3/2}} = 0.005;\qquad \frac{\overline{u_1 u_2}}{(\bar{u}_1^2\cdot\bar{u}_2^2)^{1/2}} < 0.01.$$

That a uniform temperature field be achieved, use was made of an electrically heated constantan 0.002 m dia wire grid with a square cell (0.04 m in size) placed downstream the turbulence generator grid at a distance of 0.1 m. In the test section the temperature generator grid ensured a uniform temperature field with mean square values of fluctuations equal to 0.2–0.3°C.

Velocity fluctuations were measured by the standard DISA thermoanemometer system involving constant-temperature 55D01 thermoanemometers, 55D10 linearizors, 55D26 auxilliary units; effective-value 55D35 voltmeters. Special compensated resistance thermometers were used to measure temperature fluctuations. Temperature sensors were manufactured of a copper-plated tungsten wire 2.0 μ in dia. The length of an uncovered sensitive element was about 1 mm. Statistical treatment of signals of thermoanemometers and resistance thermometers was performed on the computer "Minsk-22". For this purpose the signals of the primary devices after filtration and amplification were fed to an eight-bit six-channel analog-to-digital converter and then to the electronic digital computer. The digitizing system is described in [11].

4. EXPERIMENTAL METHODS

As expressions (2.15) and (2.17) give third-order derivatives of correlation functions $\overline{u_j u_i u_i'}$ and $\overline{u_i t t'}$ in all directions ($k = 1, 2, 3$) and at arbitrary values of i and j ($i = 1, 2, 3$; $j = 1, 2, 3$) should be calculated to find diffusional coefficients of velocity and temperature fields $S_{u\,\mathrm{dif}}$ and $S_{t\,\mathrm{dif}}$. It is quite clear that this problem cannot be practically solved. That the number of the functions measured be diminished, use was made of the approximate expressions involving homogeneous non-isotropic correlations for near points [12]

$$\overline{u_i u_j u_k'} = \frac{1}{18\sqrt{3}} q^3 R^{(3)}[(r^2 K_{ij} + K_{im}\xi_m \xi_j + K_{jm}\xi_m \xi_i)\xi_k - \tfrac{5}{2} r^2 (R_{ik}\xi_j + K_{jk}\xi_i) + \ldots]$$

$$\overline{u_i t t'} = \frac{q\bar{t}^2}{\sqrt{3}} (\tfrac{1}{6} R^{(1)} K_{mn} \xi_m \xi_n + \ldots)\xi_i$$

where

$$R^{(3)} = \frac{3\sqrt{3}}{35} \cdot \frac{1}{q^3} \left(\frac{\partial}{\partial \xi_s} \Delta_\xi \overline{u_s u_t u_t'} \right)_{\xi = 0}$$

$$R^{(1)} = \frac{\sqrt{3}}{5} \cdot \frac{1}{q\bar{t}^2} \cdot \left(\frac{\partial}{\partial \xi_s} \Delta_\xi \overline{u_s t t'} \right)_{\xi = 0}$$

$$r^2 = \xi_i \xi_i .$$

These relations show that the complexes $R^{(3)}$ and $R^{(1)}$ being in expressions (2.15) and (2.17) for diffusional coefficients, may be determined at any convenient set of i, j and k indices. Thus, the diffusional coefficients may be presented, in particular, as:

$$S_{u\,\mathrm{dif}} = 25\sqrt{3} \frac{q^4}{\nu u_1^2} \frac{\left| \left(\frac{\partial}{\partial \xi_1^3} \overline{u_1 u_1 u_1'} \right)_{\xi_1 = 0} \right|}{(-\Delta_\xi \overline{u_i u_i'})^2_{\xi = 0}}$$

$$S_{t\,\mathrm{dif}} = 36\sqrt{3} \frac{1}{æ} \frac{q^2 \bar{t}^2}{u_1^2} \frac{\left| \left(\frac{\partial}{\partial \xi_1^3} \overline{u_1 t t'} \right)_{\xi = 0} \right|}{(-\Delta_\xi \overline{t t'})^2_{\xi = 0}} .$$

So, that $S_{u\,\mathrm{dif}}$ and $S_{t\,\mathrm{dif}}$ be found experimentally, besides determination of double correlations for velocity and temperature $\overline{u_i u_i'}$ and $\overline{t t'}$, measurement should be made only of one component of the third-rank tensor of velocity correlations $\overline{u_1 u_1 u_1'}(\xi)$ and one component of the first-rank tensor of a mutual velocity and temperature correlation $\overline{u_1 t t'}(\xi)$.

When testing, to avoid multiple calibration of the apparatuses, not correlation functions but correlation coefficients were measured. To calculate longitudinal correlation coefficients (with respect to ξ_1), the "frozen turbulence" hypothesis was employed, i.e. spatial-time correlation coefficients were measured instead of spatial ones. In this case the longitudinal coordinate step was prescribed by the sampling frequency:

$$\Delta\xi_1 = \frac{u_1}{f} = u_1 \Delta\tau$$

where f is the sampling frequency, τ is the time shift.

To measure transverse correlation coefficients (with respect to ξ_2 and ξ_3), sensors were located at two spatial points, one of them being travelled in a given transverse direction. The initial distance between these sensors was measured by a microscope.

Determination of two-point mutual correlation functions for temperature and velocity is concerned with great difficulties since in this case the output signal of a thermoanemometer depends both on velocity and temperature fluctuations. The methods to measure coefficients of mutual correlation will be considered using the coefficient of triple mixed correlation $\overline{u_1 t t}(\tau)$ as an example. Output signals of the thermoanemometer and resistance thermometer, whose sensors are located close to each other, are of the form:

$$l_1 = \alpha u_1 - \beta t + m; \quad l_2 = \gamma t + k; \quad l_3 = \alpha u_1 + m$$

where e_1 is the thermoanemometer signal in the non-isothermal flow; α, β, γ, are sensitivity coefficients; m and k are thermoanemometer and resistance thermometer noises, respectively. Once the signal and noises obtained are uncorrelated, the correlation coefficient measured admits the form:

$$\rho_m = \rho_{u_1 tt(\tau)} \frac{\alpha\gamma^2 (\bar{u}_1^2 \cdot \bar{t}^2)^{1/2}}{(\bar{l}_1^2 \cdot \bar{l}_2^2)^{1/2}} - \rho_{t^2 t(\tau)} \frac{\beta\gamma^2 (\bar{t}^2)^{3/2}}{(\bar{l}_1^2)^{1/2} \bar{l}_2^2}$$

$$= \rho_{u_1 tt(\tau)} \frac{(\bar{l}_3^2 - \bar{m}^2)^{1/2} (\bar{l}_2^2 - \bar{K}^2)}{(\bar{l}_1^2)^{1/2} \bar{l}_2^2} - \rho_{t^2 t(\tau)} \frac{\left(1 - \frac{\bar{K}^2}{\bar{l}_2^2}\right) \beta (\bar{t}^2)^{1/2}}{(\bar{l}_1^2)^{1/2}}$$

$$\rho_{u_1 tt(\tau)} = \rho_m \left(\frac{\bar{l}_1^2}{\bar{l}_3^2} \right)^{1/2} + \rho_{t^2 t(\tau)} \left(\frac{\bar{l}_1^2}{\bar{l}_3^2} - 1 \right)^{1/2} .$$

Since the instrument noises make a small contribution to the integrands, then their mean-square values may be neglected. The previous relation may then be written as:

$$\rho_{u_1 tt(\tau)} = \rho_m \frac{\bar{l}_1^2}{\bar{l}_3^2} + \rho_{t^2 t(\tau)} \cdot \left(\frac{\bar{l}_1^2}{\bar{l}_3^2} - 1 \right). \tag{4.1}$$

To calculate the correlation coefficients $\rho_{u_1 tt(\tau)}$, the additional measurement was made of the correlation coefficient $\rho_{t^2 t(\tau)}$ along with statistical treatment only of the resistance-thermometer signal. Thus, that this function $\rho_{t^2 t(\tau)}$ would correspond to the function from equation (4.1), the thermometer and resistance thermometer should equally reproduce a temperature fluctuation spectrum; in this case the degree of fluctuation spectrum reproduction was estimated by a coincidence of the functions $\rho_{tt(\tau)}$ measured by the thermoanemometer and resistance thermometer. At last, as the correlations $\rho_{u_1 tt(\tau)}$ were measured by the thermoanemometer and resistance thermometer with different frequency and phase responses, then the latter should be equalized.

Two sensors were placed at the working point with coordinates $(0, 0, 0)$ to measure a mutual moment $\rho_{u_1 tt(\tau)}$. One of the sensors is operated by the constant-temperature anemometer system and is sensitive to velocity and temperature fluctuations. The resistance thermometer system is provided with the second sensor which is sensitive to temperature fluctuations. Output signals of the thermoanemometer and resistance thermometer were amplified, filtered, converted to a digital code and input to the electronic digital computer to calculate a moment $\rho_{u_1 tt(\,)}$.

The derivatives of the correlation functions measured were calculated by the finite-difference formulae. Information content, by which the correlation functions were calculated, was chosen so that the calculation error of a derivative due to statistical one of coordinate determination should be less 5%.

5. RESULTS AND DISCUSSION

Figure 1 gives a plot of mean square values of transverse and longitudinal velocity fluctuations as well as total kinetic turbulence energy against an averaged velocity. This chart shows the degree of flow unisotropy and turbulence intensity in the test section at different values of the flow velocity.

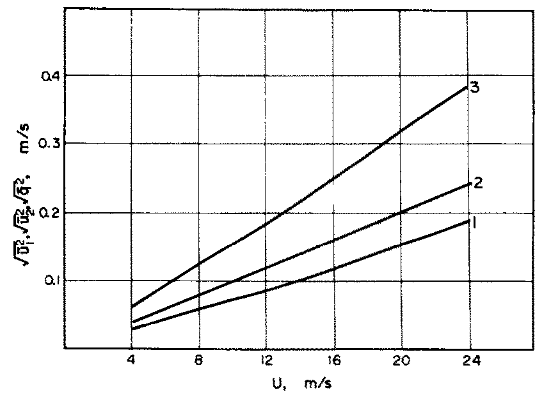

FIG. 1. Plot of mean square values of longitudinal, transverse velocity fluctuations and kinetic turbulent energy vs flow velocity 1, $\sqrt{(\bar{u}_1^2)}$; 2, $\sqrt{(\bar{u}_2^2)}$; 3, $q^2 = \sqrt{(u_i^2)}$.

In the experiment the directly measured functions used for calculating dimensionless coefficients $S_{u\mathrm{dis}}$, $S_{u\mathrm{dif}}$, $S_{t\mathrm{dis}}$ and $S_{t\mathrm{dif}}$ are normalized two-point correlation functions of velocity and temperature. Figures 2–4 show values of the correlation coefficients $\rho_{u_1u_1'(\xi_1,\xi_2)}$, $\rho_{u_2u_2'(\xi_1,\xi_2)}$ and $\rho_{tt'(\xi_1,\xi_2)}$ near a point $\xi = 0$. These functions are symmetrical, relative to the ordinate axis, that points to flow homogeneity for second-order moments.

As has been shown in Section 1, the Reynolds and Peclet numbers constructed by the appropriate scales are characteristic parameters of nonisothermal uniform turbulence. Small-scale homogeneous nonisothermal turbulence is characterized by R_λ and P_λ plotted by dissipation scales

$$\lambda_u = \frac{q}{(D_u)^{1/2}}; \quad \lambda_t = \left(\frac{\bar{t}^2}{D_t}\right)^{1/2} \tag{5.1}$$

and large-scale turbulence, by R_L and P_L constructed by degeneration scales

$$L_u = \frac{q^3}{\nu D_u}; \quad L_t = \frac{q\bar{t}^2}{æ D_t}. \tag{5.2}$$

Ranges of micro- and macroscale Reynolds and Peclet numbers corresponding to the experiment under consideration are shown in Figs. 5–6. Values of D_u and D_t functions for velocity and temperature fields were found on the basis of the measured correlation functions differentiated with respect to the coordinates ξ_1 and ξ_2. Figure 7 gives a chart of the vorticity function change for a velocity field. As is seen from this figure, with the velocity increase there occurs a very rapid growth of D_u which exceeds that by 3–4 orders. The values of D_u and D_t as well as mean square values of velocity and temperature fluctuations q and $\bar{t}^2$ were used to calculate degeneration and dissipation scales. The character of a change in λ_u and λ_t is shown in Fig. 8. Due to a very rapid increase in D_u and D_t the length scales are the decreasing functions of a mean velocity U. It is interesting to note that the ratio of dissipation scale squares is practically constant, that follows from exact asymptotic solutions. This fact confirms indirectly the validity of the adopted generalization of scales to the case of homogeneous turbulence [equation (5.1)]. Figure 9 shows a dependence of degeneration scales on the flow velocity. With an increase in U the degeneration scales tend to constant asymptotic values, and their ratio remains practically constant. This result is known as a consequence of the laws of isotropic turbulence decay at very large values of the Reynolds and Peclet numbers and confirms the validity of generalizations (5.2) to homogeneous turbulence.

That dimensionless coefficients of a homogeneous field be calculated, besides spatial two-point correlation coefficients of a velocity and temperature, measurement was made of mutual correlation coefficients $\rho_{u_1t(\xi_1)}$, $\rho_{u_1tt(\xi_1)}$ and third moments of a velocity field $\rho_{u_1u_1u_1(\xi_1)}$. The signal u_1 in a digital form was input to the computer to calculate the third moment of a velocity field, and the value of $\rho_{u_1u_1u_1(\xi_1)}$ was evaluated at four equidistant points with coordinates (0, 0, 0), ($\xi_1 = h_1$, 0, 0), ($\xi_1 = h_2$, 0, 0), ($\xi_1 = h_3$, 0, 0). The third moment calculated in such a fashion is shown in Fig. 10. Figure 11 gives a general form of a mutual moment $\rho_{u_1tt(\tau)}$ at a flow velocity of 8 m/s. It should be noted that the general form of the third moments agrees well with the data of other authors, i.e. the moments are antisymmetrical functions and approach the abscissa axis with a zero derivative (Figs. 10–11).

After calculating the appropriate differential operators of spatial two-point correlations for velocity and temperature fields at a point $\xi = 0$, statistical coefficients $S_{u\mathrm{dis}}$, $S_{t\mathrm{dis}}$, $S_{u\mathrm{dif}}$, $S_{t\mathrm{dif}}$ were calculated over a considered range of the Reynolds and Peclet numbers (Figs. 5 and 6). The character of the behaviour of coefficients $S_{u\mathrm{dis}}$ and $S_{t\mathrm{dis}}$ is shown in Fig. 12. The curves presented here are the averaging results obtained in different experiments. The necessity of such averaging is explained by the fact that calculation of derivatives due to "experimental noise" is inevitably connected with the errors even at small deviations of the determined correlation functions from the actual ones. Estimates of mean square errors of calculation of the coefficients $S_{u\mathrm{dis}}$ and $S_{t\mathrm{dis}}$ are 30 and 12%, respectively. Convenience of application of a mean square error as a numerical expression for error results in the fact that a certain confidence probability equal to 0.68 corresponds to this quantity. The confidence probability equal to 0.95 satisfies a doubled value of

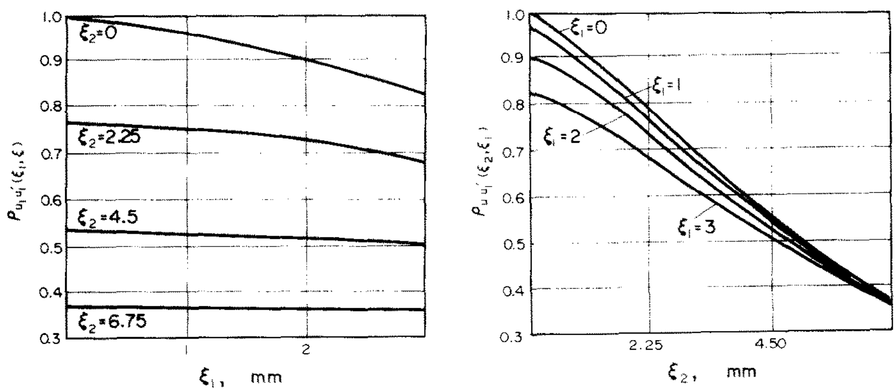

FIG. 2. Space normalized correlation functions of a longitudinal component of velocity fluctuations.

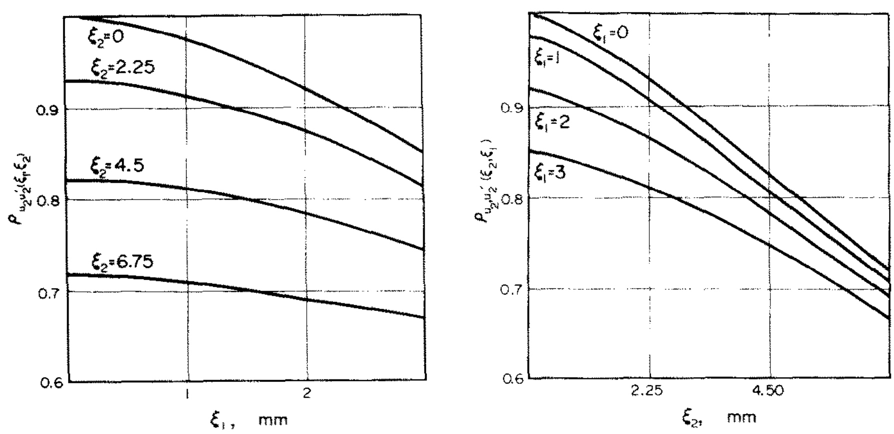

FIG. 3. Space normalized correlation functions of a transverse component of velocity fluctuations.

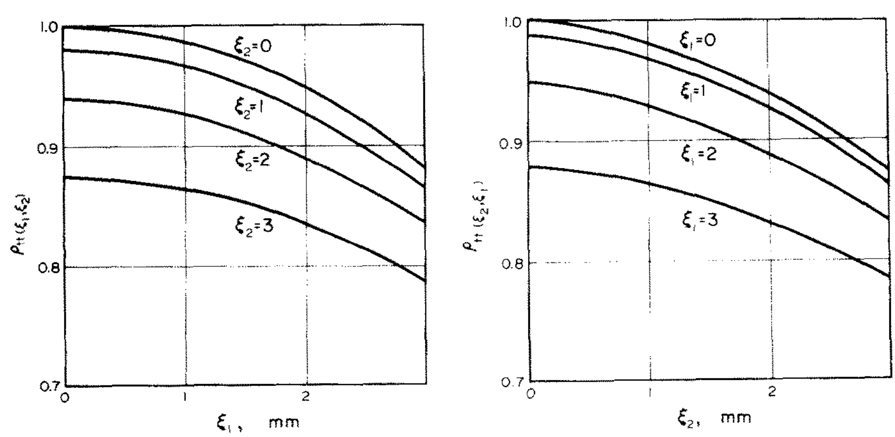

FIG. 4. Space normalized correlation functions of temperature fluctuations.

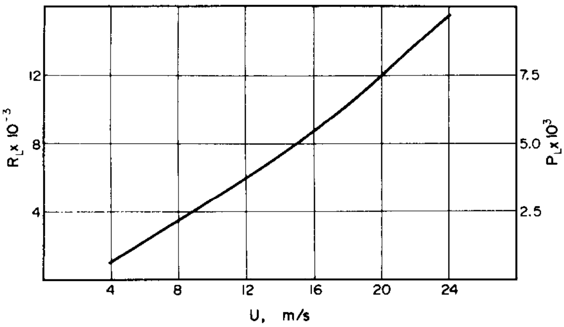

FIG. 5. Ranges of the Reynolds and Peclet numbers based on macroscales.

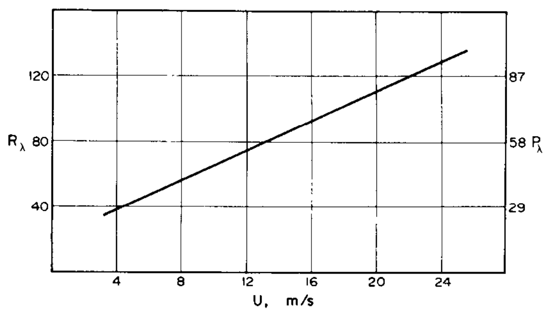

FIG. 6. Ranges of the Reynolds and Peclet numbers based on microscales.

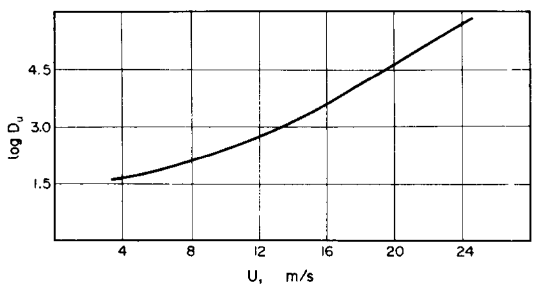

FIG. 7. Plot of functions D_u and D_t vs flow velocity.

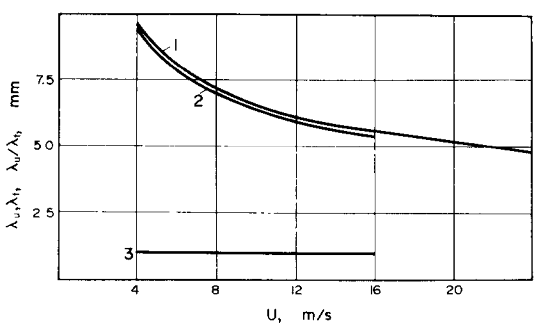

FIG. 8. Plot of velocity, temperature microscales and their ratio vs flow velocity: 1, λ_u; 2, λ_t; 3, λ_u/λ_t.

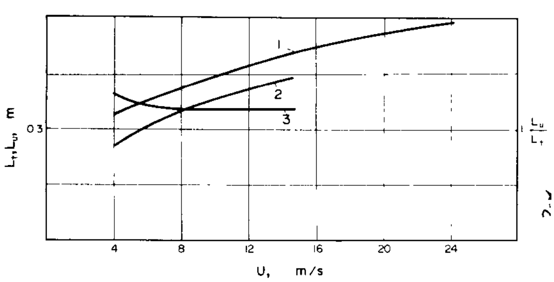

FIG. 9. Plot of velocity, temperature macroscales and their ratio vs flow velocity: 1, L_u; 2, L_t; 3, L_u/L_t.

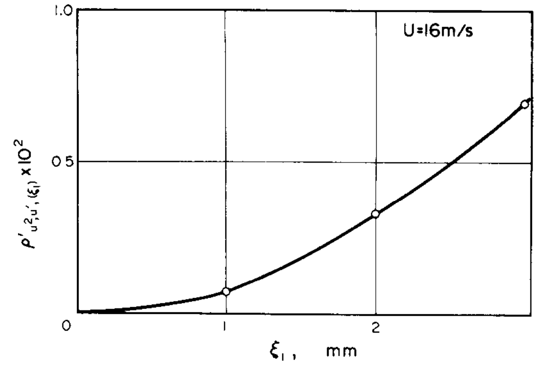

FIG. 10. Normalized function of triple two-point correlation of longitudinal velocity fluctuations: $U = 16$ m/s.

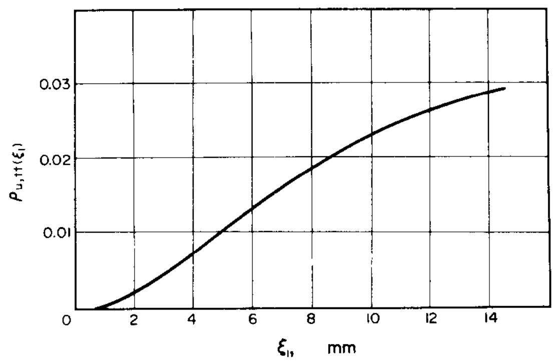

FIG. 11. Mutual normalized function of triple two-point correlation: $U = 8$ m/s.

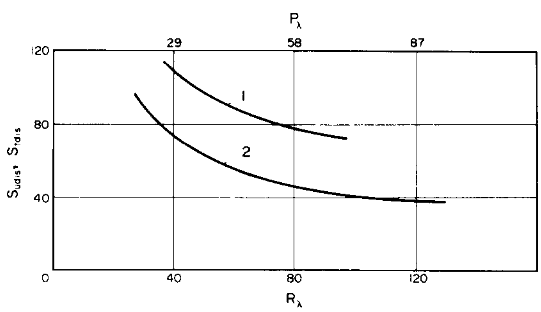

FIG. 12. Plot of coefficients $S_{u\,\text{dis}}$ (2) and $S_{t\,\text{dis}}$ (1) vs the Reynolds and Peclet numbers.

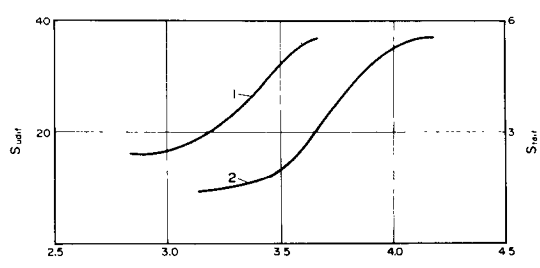

FIG. 13. Plot of coefficients $S_{u\text{dif}}$ and $S_{t\text{dif}}$ vs the Reynolds and Peclet numbers.

the mean square error. The curves depicted in Fig. 12 show that the coefficients $S_{u\,\mathrm{dis}}$ and $S_{t\,\mathrm{dis}}$ are the functions of R_L and P_L. Otherwise, over a wide range of the Reynolds and Peclet numbers the above coefficients are not universal constants. Estimates of mean square errors of these coefficients are 25 and 12%, respectively, and are also the functions of the Reynolds and Peclet numbers. These dependences are given in Fig. 13.

Taking into account statistical coefficients of scale parameters, the following functions

$$S_{u\,\mathrm{dis}} - S_{u\,\mathrm{dif}} = F_1(R_L)$$

$$\left(\tfrac{2}{3}S_{t\,\mathrm{dis}} - S_{t\,\mathrm{dif}} - \sqrt{3}\,\frac{1}{Pr}\,\frac{P_L}{R_L}\right) = F_2(P_L)$$

may be constructed, which, as it follows from asymptotic relations (2.12) and (2.13) at $R_L \gg 1$ and $P_L \gg 1$, are constants. The first of these dependences is presented in Fig. 14. The limiting value of the function equal to $(51\sqrt{3})/7$ is shown by a dotted line. As we see, at nonisotropic turbulence the quantity $F_1(R_L)$ is rather a strongly varying function of the Reynolds number. As far as R_L increases, it asymptotically approaches its limiting value. Thus, based on the experimental results obtained, an assumption may be made that at homogeneous nonisotropic turbulence, relation (2.12) is satisfied with $R_L \to \infty$. Numerical values of the function $F_2(P_L)$ depending on the macroscale Peclet number are given in Fig. 15, where the asymptotic value of F_2 obtained analytically and equal to $(16\sqrt{3})/5$ is also presented. As is seen from this chart, unlike the quickly varying function $F_1(R_L)$, the function $F_2(P_L)$ varies rather slowly. In this case it may be assumed that $F_2(P_L)$ exhibits a tendency to asymptotic value approach but its achievement is probably possible at rather large Peclet numbers than those attained in this experiment. It should be noted however that it is hardly possible to attain very large Peclet numbers for decaying turbulence under laboratory conditions. Based on the above experimental data an assumption will be therefore made that relation (2.13) is valid at very large Peclet numbers in the case of uniform homogeneous nonisotropic turbulence.

Thus, as a result of the above experimental study of a fine structure of uniform turbulence, invariants (2.12) and (2.13) for statistical coefficients of isotropic fields of a velocity and temperature are shown to be also valid for homogeneous and nonisotropic fields with very large values of the Reynolds and Peclet numbers.

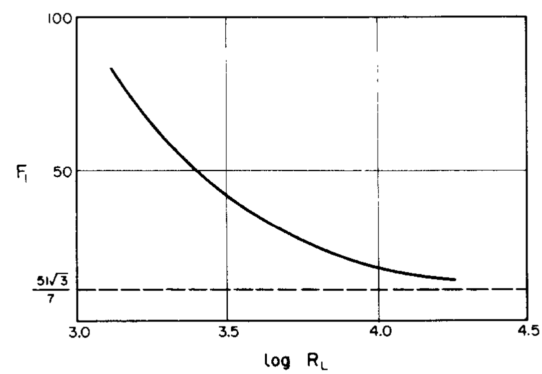

FIG. 14. Plot of the function F_1 vs the Reynolds number.

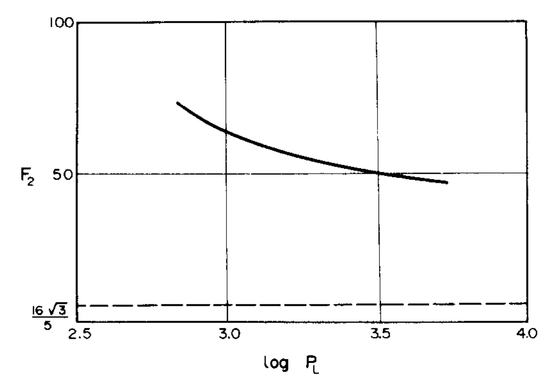

FIG. 15. Plot of the function F_2 vs the Peclet number.

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COEFFICIENTS STATISTIQUES DE TRANSFERT DANS UN CHAMP HOMOGENE DE VITESSE ET DE TEMPERATURE

Résumé—On considère la décroissance de la turbulence dans des champs de vitesse et de température homogènes et anisotropes. L'étude s'appuie sur les équations des moyennes quadratiques des fluctuations de vitesse et de température et sur les équations modélisées du champ de vorticité approprié. Les relations asymptotiques ont été vérifiées expérimentalement.

STATISTISCHE ÜBERGANGSKOEFFIZIENTEN FÜR HOMOGENE GESCHWINDIGKEITS- UND TEMPERATURFELDER

Zusammenfassung—Aufgrund der Gleichungen für mittlere Geschwindigkeits- und Temperaturschwankungen und des Modells eines Wirbelfelds werden Betrachtungen angestellt über den Abbau homogener und nicht-isotroper Geschwindigkeits- und Temperaturfelder. Asymptotische Beziehungen wurden experimentell nachgeprüft.

СТАТИСТИЧЕСКИЕ КОЭФФИЦИЕНТЫ ПЕРЕНОСА ОДНОРОДНЫХ ПОЛЕЙ СКОРОСТИ И ТЕМПЕРАТУРЫ

Аннотация — На основании уравнений для средних квадратов скорости и температуры и модельных уравнений завихренности соответствующих полей рассматривается вырождение однородных и неизотропных полей скорости и температуры. Асимптотические соотношения проверялись экспериментально.